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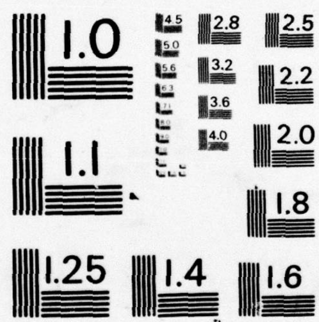

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Spectral Signal Set Extraction

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ABSTRACT. A technique is proposed for the extraction of spectral signals for which the components within a signal set are characterized by complex envelopes which exhibit correlation. The technique is based on an orthogonal decomposition of the cross (discrete) frequency correlation matrix describing the set in the presence of additive uncorrelated noise. Certain advantages of the technique relative to those based on Fourier analysis with either estimated power spectrum or minimum variance processing are presented. Specifically, it is illustrated how the detection performance of the technique is established by the total energy in the spectral set rather than by the levels of individual spectral elements in the set. Furthermore, it is shown how minimum variance spectrum analysis actually can suppress components in such a set.

I. INTRODUCTION

In many spectral signal detection systems the detector implementation embodies a model for the signal which assumes that the signal is characterized by a random phase parameter. Furthermore, it is typically assumed that the behavior of amplitude and phase of one spectral component is uncorrelated with the amplitude and phase of a spectral component which is occurring simultaneously in another region of the analysis band. As such, phase information is ignored and detection is performed on the basis of estimated power spectrum. More recently [1, 2, 3] sequential adaptive

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schemes have been considered which exploit stationary phase estimators which yield coherent detection algorithms that exhibit improved performance. However, very little consideration has been given to exploiting potential correlation between the complex envelopes of multiple spectral components with the objective of improving the multiple component extraction process. Intuitively, one feels that the best set extraction scheme should perform simultaneous coherent detection and component relating. In the following section, such an approach to coherent component relation which occurs simultaneous with detection is presented. The proposed scheme is based on a principle component analysis of the random Fourier transformed data vector (for example see [4]). Specifically, an orthogonal decomposition of the cross frequency correlation (CFC) matrix into its eigenvectors indicates that the frequency information on spectral components with coherent envelopes appears in a single eigenvector. Furthermore, the corresponding eigenvalue contains energy from all spectral components. Accordingly, it is proposed that spectral sets be extracted by identification of the appropriate eigenvalues and eigenvectors.

II. THE CROSS FREQUENCY CORRELATION (CFC) MATRIX

Let the deterministic complex N-dimensional vector $\underline{E}_k$ define the position (frequency) of a spectral component in an N-dimensional complex sample space. A random premultiplier s_k, termed the complex envelope, accounts for identical multiplicative random amplitude disturbances and additive phase perturbations on each element of the position vector $\underline{E}_k$. Consider an observed N-dimensional random data vector $\underline{X}$ consisting of $K \leq N$ spectral components and additive noise

$$\underline{X} = \sum_{k=1}^{K} s_k \underline{E}_k + \underline{N} \tag{1}$$

where $\underline{N}$ is a sample vector from an identically distributed and statistically independent vector random process with zero mean and variance σ_0^2. If the matrix E with kth column $\underline{E}_k$ and the matrix P with i-jth element $p_{ij} = \overline{s_i s_j^*}$ are introduced, then the CFC matrix

$$R = \overline{\underline{X}\,\underline{X}^*} \tag{2}$$

$$= EPE^* + \sigma_0^2 I_N \tag{3}$$

is defined wherein ( )* is the matrix complex conjugate transpose operator and I_N is an N by N identity matrix. Assuming that the vectors $\underline{E}_k$ are linearly independent a matrix Φ can be found such that

$$E = \Phi B^{-1} \tag{4}$$

where Φ has the property $\Phi^*\Phi = I_K$ and B is a K-dimensional upper triangular matrix. The matrix $B^{-1}\ P\ B^{-1*}$ is now written in diagonal form as

$$B^{-1}\ P\ B^{-1*} = \gamma\Sigma\gamma^* \tag{5}$$

where γ is a K by K matrix consisting of the orthonormal eigenvectors of $B^{-1}PB^{-1*}$ and Σ is a K by K diagonal matrix consisting of the eigenvalues $\sigma_1^2 \geq \sigma_2^2 \geq \cdots \sigma_K^2$ of $B^{-1}PB^{-1*}$.

Using (4) and (5) in (3) and post-multiplying by $\Phi\gamma$ yields

$$R\Phi\gamma = \Phi\gamma\lambda_K \tag{6}$$

where

$$\lambda_K = \Sigma + \sigma_0^2\ I_K \tag{7}$$

and

$$M = \Phi\gamma \tag{8}$$

$$= EB\gamma \tag{9}$$

give the first K eigenvalues and corresponding eigenvectors of the CFC matrix. All remaining (N-K) eigenvalues are identically σ_0^2. Equation (9) indicates that the eigenvectors $\underline{M}_\ell$ of R (columns of M) are linear combinations of the spectral position vectors $\underline{E}_k$ where the combination coefficients are functions of (a) the inner products of the position vectors $\underline{E}_i^*\underline{E}_j$ through B and (b) the complex envelope factor correlation characteristics through γ.

To realize the significance of (7) and (9) consider the case of K spectral components with orthogonal position vectors, i.e.

$$\underline{E}_i^*\ \underline{E}_j = G_i\ \delta_{ij} \quad (\delta_{ij} = \text{dirac delta}) \tag{10}$$

and complex envelope power

$$S_i = \overline{|s_i|^2}. \tag{11}$$

Let the K spectral components consist of L sets where the ℓth set $\{i \in \Omega_\ell\}$ contains components for which the magnitude squared coherence

$$|C_{ij}|^2 = \frac{|\overline{s_i\, s_j^*}|^2}{\overline{|s_i|^2}\;\overline{|s_j|^2}} = \begin{cases} 1 \text{ for } i, j \in \Omega_\ell \\ 0 \text{ for } i \in \Omega_\ell \text{ and } j \in \overline{\Omega}_\ell \end{cases}, \quad (12)$$

that is, envelope coherence exists only for components within a set. It can be shown from (7) and (9) that

$$\lambda_\ell = \sum_{i \in \Omega_\ell} G_i\, S_i + \sigma_0^2$$

$$\underline{M}_\ell = \frac{1}{\left[\sum_{i \in \Omega_\ell} G_i\, S_i\right]^{\frac{1}{2}}} \sum_{i \in \Omega_\ell} (G_i\, S_i)^{\frac{1}{2}}\, \underline{E}_i \text{ for } \ell \leq L \quad (13)$$

and

$$\lambda_\ell = \sigma_0^2 \quad \text{for} \quad N\text{-}L < \ell \leq N\;. \quad (14)$$

Thus, detecting the ℓth spectral set is accomplished by comparing an estimated λ_ℓ to σ_0^2 and relating components within the ℓth spectral set is accomplished by interrogating $\underline{M}_\ell$ to determine which components of the set $\{\underline{E}_k\}_{k=1}^K$ are present in the linear combination $\underline{M}_\ell$. The above case pertains to the situation where each of the spectral sets is disjoint, i.e., $\Omega_k \cap \Omega_\ell = 0$ with $\ell \neq k$. For the case $\Omega_k \cap \Omega_\ell \neq 0$, which includes partially coherent complex envelopes, leakage between sets occurs. However, inclusion of a spectral component within a given set will tend to be in proportion to the amount of coherence which exists between that component and the components within the set.

III. THE RELATION BETWEEN ORTHOGONAL CROSS-FREQUENCY-CORRELATION (CFC) MATRIX DECOMPOSITION AND MINIMUM VARIANCE SPECTRUM ANALYSIS

A minimum variance estimate, y_{MV}, of the complex envelope for a possible component of the random data vector $\underline{X}$ with spectral position vector $\underline{D}$ requires the filter

$$\underline{H}_{MV} = (G/\underline{D}^* R^{-1} \underline{D})\, R^{-1}\underline{D} \quad (15)$$

where $\underline{D}^*\underline{D} = G$ (see [7] for example). The expected output power for the minimum variance filter is

$$\overline{|y_{MV}|^2} = (G^2/\underline{D}^* R^{-1} \underline{D})\;, \quad (16)$$

whereas the expected output power for a conventional filtering

operation is

$$\overline{|y_C|^2} = \underline{D}^*R\underline{D} \quad . \tag{17}$$

In the preceeding section it was shown that the CFC matrix can be written as

$$R = \sum_{\ell=1}^{L} \sigma_\ell^2 \, \underline{M}_\ell \underline{M}_\ell^* + \sigma_0^2 \, I_N \quad . \tag{18}$$

Using (18) in (16) and (17) gives

$$\overline{|y_{MV}|^2} = G^2 \, \sigma_0^2/(G - \sum_{\ell=1}^{L} \psi_\ell |\underline{D}^*\underline{M}_\ell|^2) \tag{19}$$

and

$$\overline{|y_C|^2} = \sum_{\ell=1}^{L} \sigma_\ell^2 \, |\underline{D}^*\underline{M}_\ell|^2 + G\sigma_0^2 \tag{20}$$

where $\psi_\ell = \sigma_\ell^2/(\sigma_\ell^2 + \sigma_0^2)$. Consider the special case of two spectral components for both coherent and incoherent envelopes. Given orthogonal position vectors with $\underline{D}^*\underline{E}_i = \alpha_i \exp(j\phi_i)$ such that for real α_i $(0 \le \alpha_i \le G)$ there results from (19)

$$\overline{|y_{MV}|^2} = \begin{cases} G\sigma_0^2/\left[1 - \dfrac{(SNR)_1\alpha_1^2}{1 + (SNR)_1} - \dfrac{(SNR)_2\alpha_2^2}{1 + (SNR)_2}\right] & \text{for } |C_{12}|^2 = 0 \\[2ex] G\sigma_0^2/\left[1 - \dfrac{1}{1 + (SNR)_1 + (SNR)_2} \right. & \\ \quad [(SNR)_1\alpha_1^2 + (SNR)_2\,\alpha_2^2 + & \\ \quad \left. 2(SNR)_1^{1/2}(SNR)_2^{1/2}\,\alpha_1\alpha_2 \, COS(\phi_1 - \phi_2)]\right] & \\ \quad \text{for } |C_{12}|^2 = 1 & \end{cases} \tag{21}$$

and from (20)

$$\overline{|y_C|^2} = \begin{cases} G\sigma_0^2 \; [(SNR)_1 \; \alpha_1^2 + (SNR)_2 \; \alpha_2^2 + 1] \text{ for } |C_{12}|^2 = 0 \\ G\sigma_0^2 \; [(SNR)_1\alpha_1^2 + (SNR)_2\alpha_2^2 + \\ \quad 2(SNR)_1(SNR)_2\alpha_1\alpha_2 \; COS(\phi_1-\phi_2) + 1] \\ \quad \text{for } |C_{12}|^2 = 1 \end{cases} \tag{22}$$

where $(SNR)_i = GS_i/\sigma_0^2$ is the post-filter signal-to-noise ratio for $\underline{D} = \underline{E}_i$ with only the ith spectral component present. It is noted from (21) for $|C_{12}|^2 = 1$ that the filter output power for $\underline{H}_{MV}$ estimating s_1 ($\alpha_1 \simeq G$ and $\alpha_2 \simeq 0$) is reduced as $(SNR)_2$ increases. This represents a suppression of the filtered signal even though $\underline{D}$ is well matched to $\underline{E}_1$. This is because the minimum variance filter assumes that s_1 and s_2 are uncorrelated. This can be seen by considering the sensitivity expression

$$\left.\frac{\partial \overline{|y_{MV}|^2}}{\partial \alpha_1^2}\right|_{\alpha_2=0} = [1 + (SNR)_1 + (SNR)_2][G\sigma_0^2(SNR)_1] / [1 + (1 - \alpha_1^2)(SNR)_1 + (SNR)_2]^2 \tag{23}$$

which is a monotonically decreasing function of $(SNR)_2$. No such signal suppression effect is present in either conventional filtering or signal set extraction using eigenvalue(-vector) component relating.

IV. CONCLUSION

A spectrum analysis technique based on an orthogonal decomposition of the cross frequency correlation matrix has been proposed and compared to both conventional linear and minimum variance spectrum analysis. Orthogonal component spectrum analysis exhibits a capability for simultaneous extraction of all spectrally disjoint components characterized by coherence between the envelopes of the spectrally disjoint components. It has been shown that minimum variance spectrum analysis for this class of signals can actually lead to worse performance than conventional analysis. Orthogonal decomposition, on the other hand, exhibits detection performance governed by the ratio of the total signal power in an envelope coherent spectral set to the noise level in a single spectral resolution cell. As such, orthogonal spectrum analysis would have application to the extraction of a limited,

but perhaps interesting, class of signals in both frequency and spatial wavenumber analysis [5, 6]. Finally, adaptive algorithms based on gradient search techniques are available for realizing the orthogonal component spectrum analysis technique described herein [7]. These algorithms will circumvent the necessity of first estimating and storing the CFC matrix with subsequent orthogonal decomposition.

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